Band spectrum regression with panel data

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Abstract

Following Pesaran (2006), it is shown that the cross sectional averages of dependent variable and regressors in a panel data model with dynamic multifactor error structure can be used as proxies for unobserved common factors in the band spectrum regression. Basing on the idea of Corbae, Ouliaris, and Phillips (2002) it is shown that even if common factors are not dynamic, band spectrum regression must be estimated as in the dynamic case due to correlation between regressors and common factors. Method is applied to price equations.

Keywords: Band spectrum regression, panel data, cross sectional dependence, dynamic factors

Contents

Acronyms

ARMA Autoregressive Moving Average.

CCE Common Correlated Effects.

CCEMG Common Correlated Effects Mean Group.

CCEP Common Correlated Effects Pooled.

CPI Consumer Price Index.

DFT Discrete Fourier Transform.

GMM Generalized Method of Moments.

IV Instrumental Variables.

OLS Ordinary Least Squares.

PC Principal Components.

PPI Producer Price Index.

VAR Vector Autoregression Model.

1 Introduction

For the last few decades there has been a growing interest on the usage of panel data for the analysis of time series properties of economic variables. For example, an important development in analysis of panel data has been the introduction of dynamic models with a lagged dependent variable among the regressors, for the analysis of the datasets where the cross section dimension is large but the time dimension is relatively small. In this way, researches attempted to see the persistent characteristics of variables over time but using also cross sectional variation. In an influential paper, the findings of Nickell (1981) showed that when a lagged dependent variable is present in a panel dataset with a short time span, the usual fixed effects estimators are not consistent, as the number of cross sections goes to infinity. This fact led to some approaches using Instrumental Variables (IV) to correct for this inconsistency. Starting with the seminal papers by Anderson and Hsiao (1981) and by Holtz-Eakin, Newey, and Rosen (1988), the dynamic panel data models became more popular in economics. These works were followed by the Generalized Method of Moments (GMM) estimators of Arellano and Bond (1991) and the system GMM estimators of Blundell and Bond (1998). These estimators found their places in almost all areas of empirical economics. Some important applications include economic growth studies, such as Caselli, Esquivel, and Lefort (1996) who showed that the standard Barro type growth regression can be written as a dynamic panel data model. Based on this observation they concluded that the earlier results on convergence rates such as the one obtained in the original paper by Barro (1991) can be highly misleading. A survey of the use of GMM estimators with special emphasis on economic growth models can be found in Bond, Hoeffler, and Temple (2001).

On the other side, besides this literature on large N, small T panels, with the increasing availability of larger panel datasets, such as Penn World Tables and World Development Indicators of World Bank, researchers have been more interested in time series properties of panel datasets. This type of analysis usually deals with the integration characteristics of datasets. The main focus of interest is, namely, unit roots and cointegration in panel data. It has been seen as an advantage to use cross sectional variation to gain efficiency, and, draw more accurate inference about unit roots and cointegration. This is mainly an idea arising from the observation that we can increase the power of unit root tests using multiple realizations of a variable. This thought led the panel data applications in economics, such as contributions to the never ending discussion on Purchasing Power Parity hypothesis (for some early applications, see Wu, 1996; Oh, 1996; Frankel and Rose, 1996; MacDonald, 1996), Fisher relationship (see Coakley, Fuertes, and Smith, 2006; Westerlund, 2008), and Feldstein-Horioka puzzle (see Coakley and Kulasi, 1997; Ho, 2002; Coakley, Fuertes, and Smith, 2006, among others). The general gains from using panel data are listed in Hsiao (2003) and Baltagi (2008). The literature on the analysis of nonstationary panel data is now growing with the view of nonlinearities, such as structural breaks in trends (see, for example Lee and Strazicich, 2003; Banerjee and Carrion-I-Silvestre, 2006).

Even though the nonstationary panel data literature is growing rapidly there has not been a high interest on some different aspects of time series analysis of panel data. As stated, nonlinearities are being considered for the correct inference on unit root and cointegration tests, however the possibility of nonlinearities is taken into consideration only in time domain and since the cyclical behaviour common to different realizations of a time series did not draw enough attention, nonlinearities in frequency domain were not considered. In the pure, stationary time series case, starting with the primary work of Hannan (1963a) researchers have been interested in the analysis of multiple regressions in the frequency domain. Related to the earlier works of Whittle (1951), this method was decomposing the time series into different frequency components by taking the Fourier transforms of variables with the aim of investigating the relationship in different frequency bands and was making no distributional assumptions on the error term of the model, other than stationarity. This paper of Hannan was followed by other important works of his, which made use of the frequency domain regression techniques. He showed that these techniques can be used for the models with measurement errors (1963b) and Hannan and Terrell (1973) applied the techniques to multiple equation systems.[1] Since, in economics some theories predict different relationships between variables in the short and long run, the estimators proposed by Hannan found a natural area of application within economics. They were used in economic applications firstly by Sims (1972) and Engle (1974) to distinguish between different time horizons. Engle showed that this method can be applied easily using any econometric package after taking the Discrete Fourier Transform (DFT) of the variables, filtering certain frequencies and inverting the resulting complex series into time domain again. He applied the tool to permanent income hypothesis which forecast a different value of marginal propensity of consumption in the long run and in the short run.[2]

However, contrary to the fact that most of the economic time series follow either deterministic or stochastic trends, these methods were assuming stationarity of the variables in the model. Even though the general treatment of the serial correlation structure of the error term was a quite powerful aspect of the method, the stationarity assumption made it difficult to apply in economics. This problem was addressed by Phillips (1991) who employed a vector error correction model for the spectral analysis of time series and first showed that the frequency domain techniques can be used to estimate the cointegrating vectors efficiently. This work further extended by Corbae, Ouliaris, and Phillips (1994) who used the cointegration vector estimation method of Phillips but employed IV techniques for the estimation of high frequency parameters. They applied the cointegrating system to permanent income hypothesis and found that results obtained by Engle (1974) can be misleading. This work was further developed by them in their following works (1997; 2002).

As mentioned before, these developments in analysis of economic time series remained as a problem of the analysis of a single realization of time series. In an early, and to the knowledge of the author, the only paper on the spectral analysis of economic panel data, Beggs (1986) discussed the problem of estimation of spectral density.[3] He developed a panel Autoregressive Moving Average (ARMA) model to explore the hidden periodicities. In the pure time series case, the only source of error correlation is the persistence over time and this can be handled using the frequency domain techniques, as indicated. However, in panel data models the error correlation can also arise from the dependence within the cross sections. Beggs modelled this cross sectional dependence using a common factor structure, namely, he introduced a random component in the model which is changing over time but has an effect on each cross section with the same order of magnitude. Using the idea of Balestra and Nerlove (1966) on random effects models he showed that we can overcome some difficulties in spectral analysis of time series such as the need of smoothing the periodogram, by using different realizations of time series. As will be discussed later, the paper by Beggs was on the

[1] For further works on estimation of distributed lag relations see Hannan (1965) and Hannan (1967).

[2] Engle, further investigated some small sample properties of the band spectrum estimators in Engle and Gardner (1976), applied the method to the price equations (1978), gave the maximum likelihood estimators for the band spectrum regression (1980a), and dealt with the hypothesis testing problem (1980b).

[3] For a work out of economics, investigating the spectral analysis of panel data with the special emphasis on discrete time series see Beckett and Diaconis (1994).

univariate case and the spectral analysis of multivariate models with explanatory variables is still an open question.

Another simplifying assumption of the model by Beggs which can be seen as a shortcoming in economic applications was about the way of dealing with the common factor affecting the cross sections. As stated, he considered only the case where common factor has the same effect on each individual of the panel. Today, there is a growing literature about the estimation of panel data models with cross sectional dependence in different forms. Besides the literature on spatial dependence where there is a natural way of measuring the distance between cross sections (see Driscoll and Kraay, 1998; Conley, 1999; Pesaran, Schuermann, and Weiner, 2004, among others) the common factor models with heterogeneous factor loadings on cross sections are becoming increasingly popular. These models can be seen as a generalisation of the so called two way fixed effects model where we use a dummy variable for each period to control for the time varying conditional expectation of the dependent variable of the model. In the case with heterogeneous factor loadings we can relax the constraint of same effect on each cross section which is more realistic in many economic applications. Also, the common factors can be stochastic and unobservable which makes the analysis more difficult. In their paper on dynamic panel data models, Holtz-Eakin, Newey, and Rosen modelled this kind of cross sectional dependence with a single unobserved common factor which has a time varying factor loading. They proposed a quasi differencing procedure to eliminate the common factor.

In the view of this idea of elimination of common factors we can divide the literature into two parts. One includes some works which treated the factor loadings as nuisance parameters and concerned only with the estimation of slope parameters of the model and the other one where some research focused on the estimation of factor loadings. This second group of papers includes Forni, Hallin, Lippi, and Reichlin (2000, 2005), Stock and Watson (2002), Bai and Ng (2002) and Bai (2003). These papers are based on large N, large T setting and concerned with the consistent estimation of dynamic factors. The other group includes Coakley, Fuertes, and Smith (2002) and Phillips and Sul (2003). Instead of focusing on the estimation of factors and associated factor loadings Coakley, Fuertes, and Smith are dealing with the problem of estimation of the parameters of explanatory variables and Phillips and Sul are discussing the estimation of an autoregressive parameter in a model where there is no exogeneous variables.

The models in these papers are restrictive in terms of these assumptions. First, contrary to their statements the Principal Components (PC) method proposed by Coakley, Fuertes, and Smith does not produce consistent estimates of the regression parameters if the explanatory variables are correlated with common factor. This was proved by Pesaran (2006) and he suggested a general framework for consistent estimation of slope parameters. The idea underlying the method by Pesaran is that the cross sectional averages of dependent and independent variables are suitable proxies of the unobservable common effects, as $N \to \infty$. Therefore, a regression model which is augmented with these averages produces consistent estimates of the individual specific slope parameters (Common Correlated Effects (CCE) estimators). He also gives the conditions for consistent estimation of cross sectional means of these parameters (Common Correlated Effects Mean Group (CCEMG) Estimators), and proposes a pooled estimator (Common Correlated Effects Pooled (CCEP) Estimators) when slope parameters are homogeneous across cross sections. These estimators, further shown to be robust in the presence of spatial dependence in the work by Pesaran and Tosetti (2011) and Kapetanios, Pesaran, and Yamagata (2011) generalized the method for the analysis of panels with nonstationary common factors.

In this paper we extend the model by Pesaran using a distributed lag structure in explanatory variables and consider dynamic unobserved common factors. Following the works by Hannan (1963a,b) and Corbae, Ouliaris, and Phillips (2002) we are making an assumption on the transfer function of the filter coefficients which is similar to the structural break assumption in the time domain and showing that the DFT's of the cross sectional averages of dependent and explanatory variables can be used in the frequency domain regression with the aim of consistent estimation of band dependent slope parameters in the model. We also show that the idea given by Corbae, Ouliaris, and Phillips (2002) on the use of Frisch-Waugh theorem for eliminating the variables of secondary interest in the time domain prior to spectral regression indicates that the estimators proposed by Pesaran are not consistent while the slope parameters are frequency dependent even if the common factors in the model are not dynamic, therefore stable across frequencies. The consistency results directly follow from the proofs given by Pesaran, asymptotic distributions of the estimators follow the asymptotic normality and independence of Fourier coefficients.

2 The Model

The model we have is a distributed lag extension of the multifactor error model of Pesaran (2006). We are supposing that the dependent variable is generated by the following heterogeneous panel data model.

$$
\begin{aligned}
y_{nt} &= \boldsymbol{\alpha}_{\boldsymbol{n}}'\boldsymbol{d_t} + \sum_{j=0}^{+\infty} \boldsymbol{\beta}_{\boldsymbol{nj}}'\boldsymbol{x_{n,t-j}} + e_{nt} \\
&= \boldsymbol{\alpha}_{\boldsymbol{n}}'\boldsymbol{d_t} + \boldsymbol{\beta_n}(\boldsymbol{L})'\boldsymbol{x_{nt}} + e_{nt}
\end{aligned} \tag{1}
$$

where y_{nt} is the tth observation on nth cross section and $n = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$. Therefore, we are considering only the balanced panel data case. In the model, the $d \times 1$ vector $\boldsymbol{d_t}$ stands for the observable, nondynamic common factor component of the dependent variable and it may contain constant, seasonal dummies etc. and also other observable stochastic variables. Thus, the usual fixed effects model for panel data is a special case of this model. The $k \times 1$ vector $\boldsymbol{x_{nt}}$ is the vector of explanatory variables and we will specify the generating mechanism for these variables in the following. In this equation $\boldsymbol{\beta_{nj}}$ is the vector of time varying parameters of our interest. In the next section, if they are random, we will be interested in estimation of the cross section averages of these parameters and, if they are homogeneous we will focus on the estimation of $\boldsymbol{\beta_j}$. We are modelling the cross section dependence with a multivariate dynamic factor structure in the errors. For nth cross section unit at time t, e_{nt} has the following form.

$$
\begin{aligned}
e_{nt} &= \sum_{j=0}^{+\infty} \boldsymbol{\gamma}_{\boldsymbol{nj}}'\boldsymbol{f_{t-j}} + \epsilon_{nt} \\
&= \boldsymbol{\gamma_n}(\boldsymbol{L})'\boldsymbol{f_t} + \epsilon_{nt}
\end{aligned} \tag{2}
$$

$\boldsymbol{f_t}$ are the $m \times 1$ vector of common factors affecting every cross section unit n at time t and $\boldsymbol{\gamma_{nj}}$ are the corresponding factor loadings. Therefore, we are considering the general case where the common factors have a different effect on each cross section unit and in addition to the model by Pesaran, we are assuming a dynamic factor structure. This model can be seen as a generalization of the common practice in application, adding time specific dummies in a panel data regression. In the model ϵ_{nt} are the individual stochastic component. Throughout the paper, we will assume that ϵ_{nt} is a mean zero, stationary time series.

We are assuming multiple explanatory variables in the model. The $k \times 1$ vector $\boldsymbol{x_{nt}}$ is the vector of explanatory variables and we will specify the generating mechanism for these variables in the following way.

$$\begin{aligned} \boldsymbol{x_{nt}} &= \boldsymbol{A'_n d_t} + \sum_{j=0}^{+\infty} \boldsymbol{\Gamma'_{nj} f_{t-j}} + \boldsymbol{v_{nt}} \\ &= \boldsymbol{A'_n d_t} + \boldsymbol{\Gamma_n(L)' f_t} + \boldsymbol{v_{nt}} \end{aligned} \tag{3}$$

Therefore, we are assuming that the regressors $\boldsymbol{x_{nt}}$ can be correlated with the observed and unobserved factors. Again, the $d \times 1$ vector $\boldsymbol{d_t}$ is the observable deterministic or stochastic component and $\boldsymbol{\Gamma_n}$ is the matrix of factor loadings. As in the previous equation $\boldsymbol{v_{nt}}$ is the individual specific stochastic component of regressors, namely the stochastic part other than the observable and unobservable common factors. Again, we will assume that this stochastic component $\boldsymbol{v_{nt}}$ is a mean zero, stationary time series.

Now let us substitute equation (2) into (1) and stack the resulting model with equation (3) and define the random variable $\boldsymbol{s_{nt}}$ in the following way.

$$\boldsymbol{s_{nt}} = \begin{pmatrix} y_{nt} \\ \boldsymbol{x_{nt}} \end{pmatrix} = \boldsymbol{B_n(L)' d_t} + \boldsymbol{C_n(L)' f_t} + \boldsymbol{u_{nt}} \tag{4}$$

where

$$\boldsymbol{B_n(L)} = \begin{pmatrix} \boldsymbol{\alpha_n} & \boldsymbol{A_n} \end{pmatrix} \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{\beta_n(L)} & \boldsymbol{I_k} \end{pmatrix}$$

$$\boldsymbol{C_n(L)} = \begin{pmatrix} \boldsymbol{\gamma_n(L)} & \boldsymbol{\Gamma_n(L)} \end{pmatrix} \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{\beta_n(L)} & \boldsymbol{I_k} \end{pmatrix}$$

$$\boldsymbol{u_{nt}} = \begin{pmatrix} \boldsymbol{\beta_n(L)' v_{nt}} + \epsilon_{nt} \\ \boldsymbol{v_{nt}} \end{pmatrix}.$$

The critical condition about the estimation of the model in Pesaran (2006) is the rank condition concerning (a static version of) the matrix $\boldsymbol{C_n(L)}$ and this rank is determined by the unobserved factor loadings. Throughout the paper, for the ease of exposition we will assume that the rank condition is satisfied for each t, $t = 1, 2, \ldots, T$.

Finally we are defining the cross sectional averages of the observed regressors and dependent variable in the equation (4). In the following section we will show that these cross sectional averages can be used as observed proxies for the unobserved common factors.

$$\boldsymbol{\bar{s}_t} = \boldsymbol{\bar{B}(L)' d_t} + \boldsymbol{\bar{C}(L)' f_t} + \boldsymbol{\bar{u}_t} \tag{5}$$

where

$$\boldsymbol{\bar{B}(L)} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{B_n(L)} \qquad \boldsymbol{\bar{C}(L)} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{C_n(L)}$$

$$\boldsymbol{\bar{u}_t} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{u_{nt}}.$$

Here, we took the equally weighted cross sectional averages for, again, ease of exposition. Pesaran considers a more general framework for these averages and shows that the consistency results concerning the proposed estimators hold for any weighting scheme satisfying a set of "Granularity Conditions" which are satisfied for equal weights. However, even though in small samples an optimum weighting scheme problem arises, for large samples it is unimportant which weights are used.

As stated by Pesaran, this model setup is general enough to capture many usual panel data models such as random coefficient model, one way and two ways fixed effects models etc. In this paper we made an extension of the model by Pesaran to give a spectral interpretation to the model parameters. Namely, we are allowing the parameters of interest to be varying over time. In this way, using Fourier transforms of the variables we will formulate a band spectrum regression. Even though it is becoming more popular using panel data for time series analysis, mainly for unit root and cointegration testing, it is not so popular to explore the cyclical components of the economic variables using panel data. In an early paper, Beggs (1986) developed some basic tools for spectral analysis of panel data. He considers a panel autoregressive moving average model with random time effects and discusses the estimation of spectral density. Since it is one of the rare papers in this sense it may worth to compare his model and the model we have in this paper. First of all, in this paper the main focus is the estimation of the time varying parameters of explanatory variables. The paper by Beggs, however, built on a univariate analysis. Also, he considers no heterogeneity in the model. In his application, he uses data for unemployment in United States. In such an application it may be appropriate to assume homogeneity across cross sections, however, in a multivariate, multicountry model it is a necessity to consider heterogeneity across units. Therefore, we are modelling this heterogeneity, too. Another source of heterogeneity in this paper is the factor structure in the errors. Beggs considers a model with random time effects which affect each cross section in the same amount. In this paper, the time varying common effects have different impact on each cross section, which is a more realistic way to model cross section dependence in a panel data.

3 Estimation

For the estimation of the parameters of interest, namely frequency dependent slope parameters and factor loadings, we will make a set of assumptions concerning the transfer functions of the filters in the models. These assumptions are standard in the band spectrum regression literature. Here, we will briefly state these assumptions for our panel data case.

Assumption 1 (Individual Specific Stochastic Components) *For each cross section unit $n = 1, 2, \dots, N$, $\boldsymbol{\xi_{nt}} = (\epsilon_{nt}, \boldsymbol{v'_{nt}})'$ are jointly stationary time series with Wold representations $\boldsymbol{\xi_{nt}} = \sum_{j=0}^{+\infty} \boldsymbol{\Phi_{nj}} \boldsymbol{\psi_{n,t-j}}$, where $\boldsymbol{\psi_{nt}} \sim IID(\mathbf{0}, \boldsymbol{\Sigma_n})$ with finite fourth moments and the coefficients $\boldsymbol{\Phi_{nj}}$ satisfy the absolutely summability condition $\sum_{j=0}^{+\infty} j^{1/2} \left\| \boldsymbol{\Phi_{nj}} \right\| < \infty$ for each $n = 1, 2, \dots, N$. The spectral density function of $\boldsymbol{\xi_{nt}}$, can be partitioned as*

$$\boldsymbol{f_{\xi_n \xi_n}}(\boldsymbol{\lambda}) = \begin{bmatrix} \boldsymbol{f_{\epsilon_n \epsilon_n}}(\boldsymbol{\lambda}) & \mathbf{0} \\ \mathbf{0} & \boldsymbol{f_{v_n v_n}}(\boldsymbol{\lambda}) \end{bmatrix}$$

for each cross section $n = 1, 2, \dots, N$ with strictly positive spectral densities, $\boldsymbol{f_{\epsilon_n \epsilon_n}}(\boldsymbol{\lambda}) > 0$ and $\boldsymbol{f_{v_n v_n}}(\boldsymbol{\lambda}) > 0$ at each frequency λ.

Assumption 2 (Common Factors) *The observed and unobserved common factors* $\boldsymbol{g_{t'}} = (\boldsymbol{d'_{t'}}, \boldsymbol{f'_{t'}})'$ *are stationary time series with Wold representations* $\boldsymbol{g_t} = \sum_{j=0}^{+\infty} \boldsymbol{\Pi_j} \boldsymbol{\varepsilon_{t-j}}$, *where* $\boldsymbol{\varepsilon_t} \sim IID(\boldsymbol{0}, \boldsymbol{\Omega})$ *with finite fourth moments and the coefficients* $\boldsymbol{\Pi_j}$ *satisfy the absolutely summability condition* $\sum_{j=0}^{+\infty} j^{1/2} \|\boldsymbol{\Pi_j}\| < \infty$. *Furthermore, they are distributed independently of the individual specific stochastic variables* ϵ_{nt} *and* $\boldsymbol{v_{nt}}$ *for all* n, t *and* t'. *Therefore, the spectral density matrices of common factors and individual specific stochastic variables can be partitioned conformably as in the previous assumption and they are strictly positive at each frequency.*

Note that the absolute summability assumptions imply that the spectral densities of the corresponding variables are bounded at each frequency and they are uniformly continuous (Brillinger, 2001, p.23). Therefore, variances of these variables are bounded as in the Assumption (2) of Pesaran.

As in Pesaran, we make the following assumptions about the expectations of unobserved factor loadings and random parameter vector. We further specify that the transfer functions of the slope parameters and factor loadings satisfy a structural break model in the frequency domain, as in Corbae, Ouliaris, and Phillips (2002). To this end, we are defining two distinct frequency bands $\mathcal{B}_P = [-\lambda_0, \lambda_0]$ and $\mathcal{B}_{P^c} = [-\pi, -\lambda_0) \cup (\lambda_0, \pi]$ for a given value $\lambda_0 > 0$.

Assumption 3 (Factor Loadings) *The unobserved factor loadings* $\boldsymbol{\gamma_{nj}}$ *and* $\boldsymbol{\Gamma_{nj}}$ *are independently and identically distributed across cross sections for each* j *with* $j = 0, 1, \ldots, +\infty$, *of the individual stochastic component* $\boldsymbol{v_{n't}}$ *of the regressors, the error term* $\epsilon_{n't}$ *and the observed and unobserved common factors* $\boldsymbol{g_t = (d'_t, f'_t)}$ *for all* n, n', *and* t. *We further assume that their roots lie outside the unit circle and, for each cross section* $n = 1, 2, \ldots, N$, *the transfer functions given by* $\boldsymbol{\gamma_n(\lambda)} = \sum_{j=0}^{+\infty} \boldsymbol{\gamma_{nj}} e^{ij\lambda}$ *and* $\boldsymbol{\Gamma_n(\lambda)} = \sum_{j=0}^{+\infty} \boldsymbol{\Gamma_{nj}} e^{ij\lambda}$ *satisfy*

$$(6) \qquad \boldsymbol{\gamma_n(\lambda)} = \boldsymbol{\gamma_n^P} \mathbb{1}_{[\lambda \in \mathcal{B}_P]} + \boldsymbol{\gamma_n^{P^c}} \mathbb{1}_{[\lambda \in \mathcal{B}_{P^c}]}$$

$$(7) \qquad \boldsymbol{\Gamma_n(\lambda)} = \boldsymbol{\Gamma_n^P} \mathbb{1}_{[\lambda \in \mathcal{B}_P]} + \boldsymbol{\Gamma_n^{P^c}} \mathbb{1}_{[\lambda \in \mathcal{B}_{P^c}]}$$

where $\boldsymbol{\gamma_n^P}$, $\boldsymbol{\gamma_n^{P^c}}$, $\boldsymbol{\Gamma_n^P}$ *and* $\boldsymbol{\Gamma_n^{P^c}}$ *have fixed means* $\boldsymbol{\gamma^P}$, $\boldsymbol{\gamma^{P^c}}$, $\boldsymbol{\Gamma^P}$ *and* $\boldsymbol{\Gamma^{P^c}}$ *respectively, with bounded Euclidean norms. They satisfy the models*

$$(8) \qquad \boldsymbol{\gamma_n^P} = \boldsymbol{\gamma^P} + \boldsymbol{\eta_{1n}}, \quad \boldsymbol{\eta_n} \sim IID(\boldsymbol{0}, \boldsymbol{\Omega_{\eta_1}}), \quad for \;\; n = 1, 2, \ldots, N$$

$$(9) \qquad \boldsymbol{\gamma_n^{P^c}} = \boldsymbol{\gamma^{P^c}} + \boldsymbol{\eta_{2n}}, \quad \boldsymbol{\eta_n} \sim IID(\boldsymbol{0}, \boldsymbol{\Omega_{\eta_2}}), \quad for \;\; n = 1, 2, \ldots, N$$

with symmetric, nonnegative definite covariance matrices with bounded Euclidean norms.

Assumption 4 (Slope Coefficients) *For each cross section* $n = 1, 2, \ldots, N$, *the transfer function of the filter* $\boldsymbol{\beta_n(L)}$ *given by* $\boldsymbol{\beta_n(\lambda)} = \sum_{j=0}^{+\infty} \boldsymbol{\beta_{nj}} e^{ij\lambda}$ *satisfy*

$$(10) \qquad \boldsymbol{\beta_n(\lambda)} = \boldsymbol{\beta_n^P} \mathbb{1}_{[\lambda \in \mathcal{B}_P]} + \boldsymbol{\beta_n^{P^c}} \mathbb{1}_{[\lambda \in \mathcal{B}_{P^c}]}$$

where $\boldsymbol{\beta_n^P}$ *and* $\boldsymbol{\beta_n^{P^c}}$, *have fixed means* $\boldsymbol{\beta^P}$, $\boldsymbol{\beta^{P^c}}$, *respectively, with bounded Euclidean norms. They satisfy the models*

$$(11) \qquad \boldsymbol{\beta_n^P} = \boldsymbol{\beta^P} + \boldsymbol{v_{1n}}, \quad \boldsymbol{v_{1n}} \sim IID(\boldsymbol{0}, \boldsymbol{\Omega_{v_1}}), \quad for \;\; n = 1, 2, \ldots, N$$

$$(12) \qquad \boldsymbol{\beta_n^{P^c}} = \boldsymbol{\beta^{P^c}} + \boldsymbol{v_{2n}}, \quad \boldsymbol{v_{2n}} \sim IID(\boldsymbol{0}, \boldsymbol{\Omega_{v_2}}), \quad for \;\; n = 1, 2, \ldots, N$$

where $\boldsymbol{v_{1n'}}$ *and* $\boldsymbol{v_{2n'}}$ *are distributed independently of* $\boldsymbol{\gamma_{nj}}$, $\boldsymbol{\Gamma_{nj}}$, $\boldsymbol{v_{nt}}$, ϵ_{nt} *and the observed and unobserved common factors* $\boldsymbol{g_t = (d'_t, f'_t)}$ *for all* n, n' *and* $(t-j)$, $j = 0, 1, \ldots, +\infty$ *with symmetric, nonnegative definite covariance matrices with bounded Euclidean norms.*

Now let us define the following matrices of observations on dependent variable y_{nt} and the regressors $\boldsymbol{x_{nt}}$. $\boldsymbol{X_n = [x_{n1}, x_{n2}, \ldots, x_{nT}]'}$ is the $T \times k$ matrix of observations on regressors and $\boldsymbol{y_n} = [y_{n1}, y_{n1}, \ldots, y_{nT}]'$ is the $T \times 1$ matrix of observations on the dependent variable for each $n = 1, 2, \ldots, N$. We define the row vector $\boldsymbol{l_k}$ as

$$\boldsymbol{l_k} = \left(1, e^{i\lambda_k}, e^{2i\lambda_k}, \ldots, e^{(T-1)i\lambda_k}\right)$$

where $\lambda_k = 2\pi k/T$ are the fundamental frequencies with the imaginary unit $i = \sqrt{-1}$. The discrete Fourier transform matrix, $\boldsymbol{L}$ is defined as

$$\boldsymbol{L} = \frac{1}{\sqrt{T}} \left(\boldsymbol{l}_0^\dagger, \boldsymbol{l}_1^\dagger, \ldots, \boldsymbol{l}_{T-1}^\dagger\right)^\dagger$$

which is a $T \times T$ unitary matrix. Here, $\dagger$ stands for the complex conjugate transpose of the matrix. The unitary matrix for transformation used in the paper will be given by $\boldsymbol{W} = \boldsymbol{EL}$, with

$$\boldsymbol{E} = \begin{pmatrix} 0 & 1 \\ \boldsymbol{I_{T-1}} & 0 \end{pmatrix}$$

which is a permutation matrix to reorder the rows of the DFT matrix $\boldsymbol{L}$ such that when we compute $\boldsymbol{Wy_n}$ for one of the cross sections, kth row of this matrix gives the DFT $w_y(\lambda_s)'$ with $s = k - 1$.

We define the $T \times T$ diagonal frequency selection matrix $\boldsymbol{P}$ as the matrix with ones on the diagonal if the corresponding frequency is included in the frequency band $\mathcal{B}_P$ and zeros otherwise. The complementary matrix, in the sense of selecting the frequencies in the band $\mathcal{B}_{P^c}$ is shown by $\boldsymbol{P^c}$. The band pass filter suggested by Engle (1974) is given by the matrix $\boldsymbol{\Psi} = \boldsymbol{W}^\dagger \boldsymbol{PW}$ which takes the time series from the time domain, eliminates the undesired frequencies and converts it into time domain again. For the frequency band $\mathcal{B}_{P^c}$ we show it as $\boldsymbol{\Psi^c}$.

To specify the assumption on the identification of the individual slope coefficients $\boldsymbol{\beta_n^P}$, $\boldsymbol{\beta_n^{P^c}}$ and their expectations $\boldsymbol{\beta^P}$ and $\boldsymbol{\beta^{P^c}}$ we will define the following residual creating matrices for the frequency band $\mathcal{B}_P$:

$$\boldsymbol{M_h = I_T - H(H'H)^- H'} \tag{13}$$

$$\boldsymbol{M_g = I_T - G(G'G)^- G'} \tag{14}$$

where $\boldsymbol{H} = (\boldsymbol{D}, \boldsymbol{\Psi\bar{S}}, \boldsymbol{\Psi^c\bar{S}})$ and $\boldsymbol{G} = (\boldsymbol{D}, \boldsymbol{\Psi F}, \boldsymbol{\Psi^c F})$ with the observation matrices $\boldsymbol{D} = (\boldsymbol{d_1, d_2, \ldots, d_T})'$, $\boldsymbol{F} = (\boldsymbol{f_1, f_2, \ldots, f_T})'$ and $\boldsymbol{\bar{S}} = (\boldsymbol{\bar{s}_1, \bar{s}_2, \ldots, \bar{s}_T})'$. We also define the matrix of observations on the observed regressors as $\boldsymbol{Z_n} = (\boldsymbol{\Psi X_n}, \boldsymbol{\Psi^c X_n})$.

Assumption 5 (Identification of $\boldsymbol{\beta_n}$) *For each cross section $n = 1, 2, \ldots, N$, the observation matrices $\boldsymbol{\Upsilon_n} = T^{-1}(\boldsymbol{Z_n' M_h Z_n})$ and $\boldsymbol{\Upsilon_{ng}} = T^{-1}(\boldsymbol{Z_n' M_g Z_n})$ are of full column rank, therefore invertible and their inverses have bounded second order moments.*

3.1 Common Correlated Effects Estimators

Common Correlated Effects estimators are built on the fact that the cross sectional averages of the observed variables in the model are suitable proxies for the unobserved common factors. Therefore, these proxies lead to a direct way to consistently estimate the slope parameters in the model. To see this let us rewrite the equation (5) in the following way.

$$\bar{\boldsymbol{C}}(\boldsymbol{L})'\boldsymbol{f_t} = \bar{\boldsymbol{s}}_{\boldsymbol{t}} - \bar{\boldsymbol{B}}(\boldsymbol{L})'\boldsymbol{d_t} - \bar{\boldsymbol{u}}_{\boldsymbol{t}} \tag{15}$$

And, under Assumption (3), this equation can be written as

$$\boldsymbol{f_t} = \bar{\boldsymbol{G}}(\boldsymbol{L})(\bar{\boldsymbol{s}}_{\boldsymbol{t}} - \bar{\boldsymbol{B}}(\boldsymbol{L})'\boldsymbol{d_t} - \bar{\boldsymbol{u}}_{\boldsymbol{t}}) \tag{16}$$

where

$$\bar{\boldsymbol{G}}(\boldsymbol{L}) = (\bar{\boldsymbol{C}}(\boldsymbol{L})\bar{\boldsymbol{C}}(\boldsymbol{L})')^{-1}\bar{\boldsymbol{C}}(\boldsymbol{L}).$$

However, under Assumption (1), we have

$$\bar{\boldsymbol{u}}_{\boldsymbol{t}} \overset{q.m.}{\to} \boldsymbol{0} \quad as \;\; N \to \infty \tag{17}$$

and, again, by the conditions on the expected values of the factor loadings given in Assumption (3)

$$\bar{\boldsymbol{C}}(\boldsymbol{\lambda}) \overset{p}{\to} \begin{cases} \bar{\boldsymbol{C}}^{\boldsymbol{P}} = \begin{pmatrix} \boldsymbol{\gamma}^{\boldsymbol{P}} & \boldsymbol{\Gamma}^{\boldsymbol{P}} \end{pmatrix} \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{\beta}^{\boldsymbol{P}} & \boldsymbol{I_k} \end{pmatrix} & \text{if } \lambda \in \mathcal{B}_P \\ \bar{\boldsymbol{C}}^{\boldsymbol{P^c}} = \begin{pmatrix} \boldsymbol{\gamma}^{\boldsymbol{P^c}} & \boldsymbol{\Gamma}^{\boldsymbol{P^c}} \end{pmatrix} \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{\beta}^{\boldsymbol{P^c}} & \boldsymbol{I_k} \end{pmatrix} & \text{if } \lambda \in \mathcal{B}_{P^c} \end{cases} \quad as \;\; N \to \infty$$

where $\bar{\boldsymbol{C}}(\boldsymbol{\lambda}) = \sum_{j=0}^{+\infty} \bar{\boldsymbol{C}}_{\boldsymbol{j}} e^{ij\lambda}$. Therefore,

$$\boldsymbol{w_f}(\boldsymbol{\lambda}) \overset{p}{\to} \begin{cases} \bar{\boldsymbol{G}}^{\boldsymbol{P}}(\boldsymbol{w_s}(\boldsymbol{\lambda}) - \bar{\boldsymbol{B}}^{\boldsymbol{P}}\boldsymbol{w_d}(\boldsymbol{\lambda})) & \text{if } \lambda \in \mathcal{B}_P \\ \bar{\boldsymbol{G}}^{\boldsymbol{P^c}}(\boldsymbol{w_s}(\boldsymbol{\lambda}) - \bar{\boldsymbol{B}}^{\boldsymbol{P^c}}\boldsymbol{w_d}(\boldsymbol{\lambda})) & \text{if } \lambda \in \mathcal{B}_{P^c} \end{cases} \quad as \;\; N \to \infty. \tag{18}$$

where $\bar{\boldsymbol{G}}^{\boldsymbol{P}} = (\bar{\boldsymbol{C}}^{\boldsymbol{P}}\bar{\boldsymbol{C}}^{\boldsymbol{P}'})^{-1}\bar{\boldsymbol{C}}^{\boldsymbol{P}}$, $\bar{\boldsymbol{B}}^{\boldsymbol{P}}$ is given by

$$\bar{\boldsymbol{B}}^{\boldsymbol{P}} = \begin{pmatrix} \boldsymbol{\alpha} & \boldsymbol{A} \end{pmatrix} \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{\beta}^{\boldsymbol{P}} & \boldsymbol{I_k} \end{pmatrix}$$

and, $\bar{\boldsymbol{G}}^{\boldsymbol{P^c}}$ and $\bar{\boldsymbol{B}}^{\boldsymbol{P^c}}$ are defined analogously. $\boldsymbol{w_a}(\boldsymbol{\lambda}) = 1/\sqrt{T}\sum_{t=1}^{T} \boldsymbol{a_t} e^{it\lambda}$, shows the DFT of a time series $\boldsymbol{a_t}$. This result shows that we can use the cross sectional averages of observable variables, $\bar{\boldsymbol{s}}_{\boldsymbol{t}}$, and $\boldsymbol{d_t}$ as observable proxies for the unobserved factors $\boldsymbol{f_t}$ in the frequency domain regression in order to estimate the frequency dependent slope parameters $\boldsymbol{\beta}_{\boldsymbol{n}}^{\boldsymbol{P}}$ and $\boldsymbol{\beta}_{\boldsymbol{n}}^{\boldsymbol{P^c}}$, and their cross sectional means $\boldsymbol{\beta}^{\boldsymbol{P}}$, $\boldsymbol{\beta}^{\boldsymbol{P^c}}$ which are given by Assumption (4).

Now, for each cross section $n = 1, 2, \ldots, N$, let us write the equation 1 in the matrix notation as follows.

$$\boldsymbol{y_n} = \boldsymbol{D}\boldsymbol{\alpha_n} + \boldsymbol{X_n}\boldsymbol{\beta_n}(\boldsymbol{L}) + \boldsymbol{F}\boldsymbol{\gamma_n}(\boldsymbol{L}) + \boldsymbol{\epsilon_n} \tag{19}$$

where $\boldsymbol{\epsilon_n} = (\epsilon_{n1}, \epsilon_{n2}, \ldots, \epsilon_{nT})'$ and with $\boldsymbol{D}$, $\boldsymbol{F}$ as defined before. In order to estimate the frequency dependent coefficients we can multiply this equation with the DFT matrix and frequency selection matrices defined before. This operation leads to the following system of equations.

$$
\begin{aligned}
\boldsymbol{PWy_n} &= \boldsymbol{PWD\alpha_n} + \boldsymbol{PWX_n\beta_n^P} + \boldsymbol{PWF\gamma_n^P} + \boldsymbol{PW\epsilon_n} \\
\boldsymbol{P^cWy_n} &= \boldsymbol{P^cWD\alpha_n} + \boldsymbol{P^cWX_n\beta_n^{P^c}} + \boldsymbol{P^cWF\gamma_n^{P^c}} + \boldsymbol{P^cW\epsilon_n}
\end{aligned} \tag{20}
$$

These equations are in complex terms. Engle (1974) suggests taking the inverse DFT of the equations by multiplying them with $W^\dagger$ and estimating them separately. Instead of doing this, we can sum the two equations and take the inverse DFT. This leads to the following equation for $\boldsymbol{y_n}$.

$$
\boldsymbol{y_n} = \boldsymbol{D\alpha_n} + \boldsymbol{\Psi X_n\beta_n^P} + \boldsymbol{\Psi^c X_n\beta_n^{P^c}} + \boldsymbol{\Psi F\gamma_n^P} + \boldsymbol{\Psi^c F\gamma_n^{P^c}} + \epsilon_n \tag{21}
$$

Therefore we obtain an equation which is written in terms of the observed dependent variable $\boldsymbol{y_n}$ and the error term ϵ_n is the one which is stated in equation 2. Following Hannan (1963a), CCE estimators for the frequency dependent vectors are given by

$$
\boldsymbol{\hat{\beta}_n^P} = \boldsymbol{(X_n'M_h\Psi M_hX_n)^{-1}(X_n'M_h\Psi M_hy_n)} \tag{22}
$$

$$
\boldsymbol{\hat{\beta}_n^{P^c}} = \boldsymbol{(X_n'M_h\Psi^c M_hX_n)^{-1}(X_n'M_h\Psi^c M_hy_n)}. \tag{23}
$$

As can be seen, for each frequency bands we are using the residual creating matrix $\boldsymbol{M_h}$ to filter the variables and obtain the corresponding band spectral CCE estimators. These estimators can be written in different but equivalent ways using the orthogonality properties of sinusoidals. Under the assumptions stated, consistency result given in Theorem 1. of Pesaran (2006) directly implies the consistency of band spectral estimators. The only difference in consistency proof is based on the convergence result (18) of this paper. This result shows that as $N \to \infty$ using the DFT of the cross sectional averages of the dependent and independent variables is equivalent to using the unobservable common factors. The critical result for the time domain estimators are given in equation (38) of Pesaran. Replacing the time domain variables with the frequency filtered variables, using (18) the consistency of estimators (22) and (23) follows the same lines. Asymptotic normality of the band spectral regression estimators are established in Corbae, Ouliaris, and Phillips (1997), Theorem 5. That is, if $\sqrt{T}/N$ as $(N,T) \xrightarrow{j} \infty$,

$$
\sqrt{T}(\boldsymbol{\hat{\beta}_n^P} - \boldsymbol{\beta_n^P}) \xrightarrow{j} N(\boldsymbol{0}, \boldsymbol{V_n}) \tag{24}
$$

with an analogous result for $\boldsymbol{\hat{\beta}_n^{P^c}}$ The standard sandwich formula for this $\boldsymbol{V_n}$ is given in equation (48) of Corbae, Ouliaris, and Phillips (2002) and it requires an initial estimate of the residual spectral density, as in the autocorrelation in the case of Pesaran. He suggests a Newey-West type estimator for the variance.

Under assumptions (3) and (4) slope coefficients and factor loadings follow a random coefficient model with fixed means. Therefore, we can adapt Pesaran's CCEMG formulae for the means of frequency dependent coefficients.

$$\hat{\boldsymbol{\beta}}^{\boldsymbol{P}}_{\boldsymbol{MG}} = N^{-1}\sum_{n=1}^{N}\hat{\boldsymbol{\beta}}^{\boldsymbol{P}}_{\boldsymbol{n}} \tag{25}$$

with analogous formula for $\hat{\boldsymbol{\beta}}^{\boldsymbol{P^c}}_{\boldsymbol{MG}}$. Pesaran shows the consistency and asymptotic normality of CCEMG estimators as $(N,T) \xrightarrow{j} \infty$ which does not require any additional assumption in our case. A consistent estimator of the variance of the CCEMG estimators is

$$\hat{\boldsymbol{\Sigma}}^{\boldsymbol{P}}_{\boldsymbol{MG}} = (N-1)^{-1}\sum_{n=1}^{N}(\hat{\boldsymbol{\beta}}^{\boldsymbol{P}}_{\boldsymbol{n}} - \hat{\boldsymbol{\beta}}^{\boldsymbol{P}}_{\boldsymbol{MG}})(\hat{\boldsymbol{\beta}}^{\boldsymbol{P}}_{\boldsymbol{n}} - \hat{\boldsymbol{\beta}}^{\boldsymbol{P}}_{\boldsymbol{MG}})' \tag{26}$$

again, with analogous formula for $\hat{\boldsymbol{\Sigma}}^{\boldsymbol{P^c}}_{\boldsymbol{MG}}$.

3.2 A Note on Frisch-Waugh Theorem

Suppose that we do not have dynamic factors as introduced in the previous section. In this case one may think that we can run the regressions extended with the cross section averages in the time domain and run the band spectrum regressions with the filtered variables. However, as shown by Corbae, Ouliaris, and Phillips (2002) in a latent variable model, Frisch-Waugh Theorem does not apply in the case of frequency dependent regression coefficients. Consider the following equation system.

$$y_{nt} = \boldsymbol{\alpha}'_{\boldsymbol{n}}\boldsymbol{d_t} + \boldsymbol{\gamma}'_{\boldsymbol{n}}\boldsymbol{f_t} + \tilde{y}_{nt} \tag{27}$$

$$\boldsymbol{x_{nt}} = \boldsymbol{A}'_{\boldsymbol{n}}\boldsymbol{d_t} + \boldsymbol{\Gamma}'_{\boldsymbol{n}}\boldsymbol{f_t} + \tilde{\boldsymbol{x}}_{\boldsymbol{nt}} \tag{28}$$

where all dependent variable and regressors are as defined before but the individual specific stochastic components are replaced with $\tilde{y}_{nt}$ and $\tilde{\boldsymbol{x}}_{\boldsymbol{nt}}$ which are satisfying the same assumptions as the error terms in the previous model. And let us specify the mechanism which is relating the dependent and independent variable as follows.

$$\tilde{y}_{nt} = \boldsymbol{\beta_n(L)'}\tilde{\boldsymbol{x}}_{\boldsymbol{nt}} + \epsilon_{nt} \tag{29}$$

Suppose that Assumption (4) on the transfer function of the filter $\boldsymbol{\beta_n(L)}$ still holds, that is it satisfies equation (10). As in the derivation of equation system (20), let us take the DFT of (29) for the bands $\mathcal{B}_P$ and $\mathcal{B}_{P^c}$ and take the inverse DFT of the sum of resulting system. We obtain the following equation.

$$\tilde{\boldsymbol{y}}_{\boldsymbol{n}} = \boldsymbol{\Psi}\tilde{\boldsymbol{X}}_n\beta^{\boldsymbol{P}}_{\boldsymbol{n}} + \boldsymbol{\Psi}^c\tilde{\boldsymbol{X}}\beta^{\boldsymbol{P^c}}_{\boldsymbol{n}} + \boldsymbol{\epsilon_n} \tag{30}$$

where $\tilde{\boldsymbol{y}}_{\boldsymbol{n}} = (\tilde{y}_{n1}, \tilde{y}_{n2}, \ldots, \tilde{y}_{nT})'$, $\tilde{\boldsymbol{X}}_{\boldsymbol{n}} = (\tilde{\boldsymbol{X}}_{\boldsymbol{n1}}, \tilde{\boldsymbol{X}}_{\boldsymbol{n2}}, \ldots, \tilde{\boldsymbol{X}}_{\boldsymbol{nT}})'$ and $\boldsymbol{\epsilon_n}$ is as defined before. Let us write the equations (27) and (28) in matrix notation as following.

$$\begin{aligned} \boldsymbol{y_n} &= \boldsymbol{D\alpha_n} + \boldsymbol{F}\gamma_{\boldsymbol{n}} + \tilde{\boldsymbol{y}}_{\boldsymbol{n}} \\ \boldsymbol{X_n} &= \boldsymbol{DA_n} + \boldsymbol{F\Gamma_n} + \tilde{\boldsymbol{X}}_{\boldsymbol{n}} \end{aligned} \tag{31}$$

Combining (30) and (31) we obtain the following model for the observed dependent variable $\boldsymbol{y_n}$.

$$\begin{aligned} \boldsymbol{y_n} =& \boldsymbol{\Psi D}(\boldsymbol{\alpha_n} - \boldsymbol{A_n}\beta^{\boldsymbol{P}}_{\boldsymbol{n}}) + \boldsymbol{\Psi}^c\boldsymbol{D}(\boldsymbol{\alpha_n} - \boldsymbol{A_n}\beta^{\boldsymbol{P^c}}_{\boldsymbol{n}}) \\ &+ \boldsymbol{\Psi F}(\gamma_{\boldsymbol{n}} - \boldsymbol{\Gamma_n}\beta^{\boldsymbol{P}}_{\boldsymbol{n}}) + \boldsymbol{\Psi}^c\boldsymbol{F}(\gamma_{\boldsymbol{n}} - \boldsymbol{\Gamma_n}\beta^{\boldsymbol{P^c}}_{\boldsymbol{n}}) \\ &+ \boldsymbol{\Psi X_n}\beta^{\boldsymbol{P}}_{\boldsymbol{n}} + \boldsymbol{\Psi}^c\boldsymbol{X_n}\beta^{\boldsymbol{P^c}}_{\boldsymbol{n}} + \boldsymbol{\epsilon_n} \end{aligned} \tag{32}$$

This relation shows explicitly that in the case of frequency dependent regression coefficients, even if the common factors are not dynamic, that is stable across frequencies, we cannot filter the variables in the time domain to apply band spectrum regression for the analysis of regression coefficients. This arises from the fact that whenever the frequency dependence assumption holds for the regression coefficients, the factor loadings and the parameters of the deterministic components are also frequency dependent. Therefore, in the latent variable case, instead of applying Frisch-Waugh theorem for passing from time domain to frequency domain, as suggested by Corbae, Ouliaris, and Phillips we have to employ deterministic components, observable common factors and the cross section averages of dependent variable and regressors explicitly in the band spectrum regression.

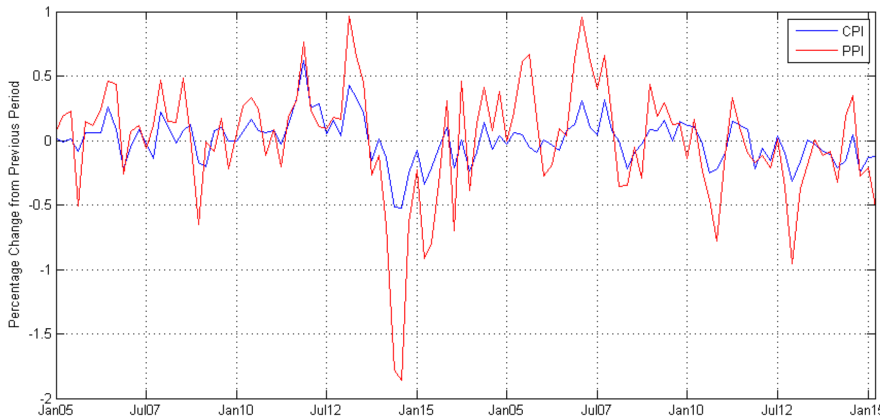
4 Application

Engle (1978) uses band spectrum regression to detect the possible parameter instabilities in price equations across frequency bands. He runs a set of dynamic models of the percentage change in Consumer Price Index (CPI) on the change in factor prices and concludes that price equations satisfy different models in different frequencies. The dynamic relationship between consumer and producer prices examined carefully in the literature. The main method of investigating the relationship is using Vector Autoregression Model (VAR) to refer to the predictive power of one indicator on the other one. Using Granger causality tests while Colclough and Lange (1982) cannot find evidence in favor of a causality running from producer prices to consumer prices, Caporale, Katsimi, and Pittis (2002) concludes that producer prices have a predictive power on consumer prices. In this section the panel data band spectrum regression method described above will be used to estimate price equations.[4] We are using monthly data to focus on both high and low frequencies and due to data availability reasons we are restricting our attention to a set of OECD countries. Data set covers the period between 02:2005 and 03:2014 for 24 countries. The summary statistics for the rate of change in CPI and Producer Price Index (PPI) are given in appendix.

The time series graph of the cross sectional averages of the variables can be seen in Figure 1. Since we are using the rate of change from the previous period rather than the same period of the previous year, both indices show a seasonal behaviour. We will further investigate this behaviour and therefore the graph is reported for deseasonalized series. Because of the high volatility in prices it is hard to make conclusion about the cyclical relationships between variables by investigating the time series graphs. However we can see that the changes in consumer prices are more stable and we can conclude about a persistent behaviour of producer prices.

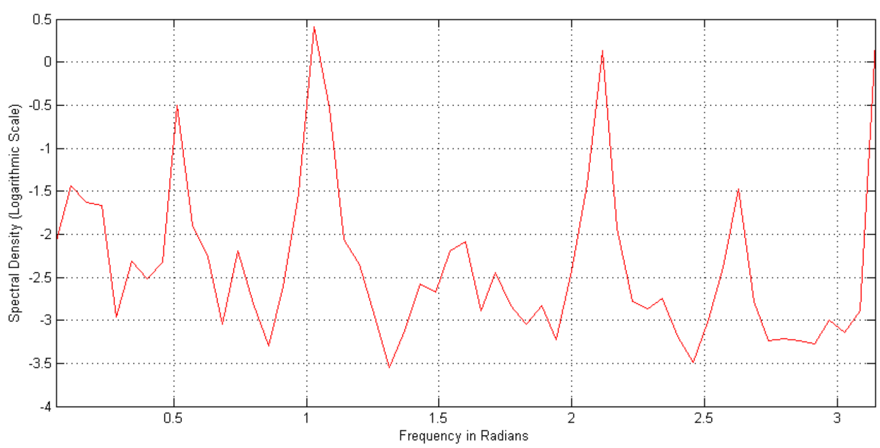
[4]Fixed effects models, panel unit root tests and cross section dependence test are computed in Stata. Other computations are performed in Matlab. Dataset and Matlab routines are available from the author.

Figure 1: Time Series Graph of Cross Sectional Averages



In order to use the tools describes above the series have to be stationary. Breitung and Das (2005) test for unit roots in cross sectionally dependent panel data shows that both series are stationary. For CPI and PPI, the test statistics are -10.356 ($p < 0.00$) and -11.255 ($p < 0.00$), respectively. A first order autoregressive fixed effects model leads to AR coefficients of 0.11 ($p < 0.00$) and 0.37 ($p < 0.00$) for the series. Therefore, we conclude that both series are stationary and the naive observation drawn from time series graphs can be verified using formal tools. We can further examine the time series properties of the variables using spectral densities. As stated before Beggs (1986) defined the basic tools for spectral analysis of panel data. Here, using a simplified version of the main method given by him, we computed the periodograms of seasonal CPI and PPI, using the information from all cross sections. The formula for is given in equation (8) of Beggs.

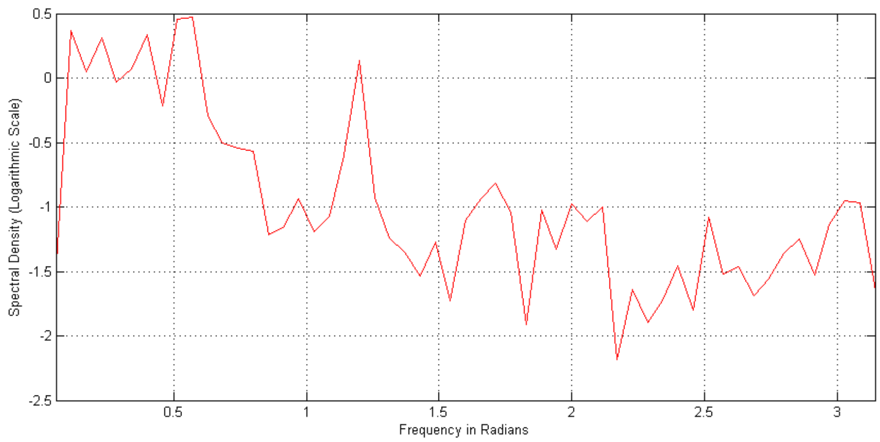
Figure 2: Average Periodogram of CPI



As can be seen from Figure 2. the periodogram of CPI shows certain peeks due to seasonality.

This behaviour is a result of a persistence from a the same months of subsequent years. This suggests an ARMA model for the series with a 12th order AR term. Also the U-shape shows that there is need for lower order AR terms. Contrary to the periodogram of CPI, the periodogram of PPI Figure 3. is (roughly) downward sloping which is a result of the persistent behaviour mentioned before. Because of the (relatively) higher first order AR parameter most of the variance of the series can be explained by low frequency movements. The comments made on the seasonal behaviour of PPI is valid in the case of PPI, too. Therefore, in our further analysis we will always use deseasonalized series.

Figure 3: Average Periodogram of PPI



As a first insight on the relationships between variables we ran individual Ordinary Least Squares (OLS) regressions for each county in the sample. These results are given in Appendix. As can be seen from the first row from the Table 5. for most of the countries there is a statistically significant relationship between variables. For some of the countries the parameter estimate is around 0.4 which indicates that a ten percentage point increase in producer prices leads to a 4 percentage point increase in consumer prices. As in the application of Engle (1978) we computed the group means of parameter estimates. This mean for OLS regressions is 0.18 which is quite lower than the usual findings in the literature, as well as Engle. However, it must be noted that our analysis is descriptive in the sense that we do not control for any other variables in these OLS regressions. To check the stability of these results over frequencies we ran the band spectrum regressions. These frequency domain results are given in the following columns of the same table. For comparison purposes we followed Engle's practice to determine the frequency bands of interest. Low frequency band include the 0-0.072 frequency interval (as a fraction of cycles) and high frequency band includes the interval 0.072-0.5. Note that Engle further extends the eliminated seasonal frequencies by two sided neighbours of seasonals and harmonics (namely, frequencies 1/12, 1/6, 1/4, 1/3, 5/12 and 1/2, as fraction of cycles). Here we are using a time domain regression prior to band spectrum regression to eliminate the seasonal movements.[5] Individual

[5] The result of Corbae, Ouliaris, and Phillips (2002) on Frisch-Waugh theorem does not make this practice invalid since eliminating these frequencies and employing monthly dummies in a time domain regression are equivalent.

frequency domain regressions show that contrary to the observation of Engle all countries but Estonia the null hypthesis of equality of low and high frequency parameters cannot be rejected. However we can still talk about an economic difference of the parameters. For example, for Slovenia and Austria, the differences between low and high frequency parameters are 0.31 and 0.28, respectively. For 6 countries the high frequency parameter is greater than the low frequency parameter. The mean of low frequency parameter is larger than time domain estimate, which is 0.22 and for the high frequencies this mean is 0.12. Again, we obtain lower parameter estimates comparing to the past literature.

Table 1: Fixed Effects Regressions

	(1)	(2)
PPI	0.126	0.112
	(11.99)	(9.94)
PPI(-1)		0.0421
		(3.73)
Constant	-0.0634	-0.0498
	(-2.05)	(-1.60)
Observations	2640	2616

Note: Dependent variable is CPI. t-ratios are given in parentheses. Both models include monthly dummies.

As stated by Engle himself, if there is cross sectional dependence between individual time series the means of the parameters which are computed here are not statistically valid for interpretation. In our further analysis we will employ the panel data techniques in the literature and the one explained above in order to control for cross sectional dependence. As a benchmark case we ran two fixed effects regressions. These results are given in Table 1. As can be seen, the fixed effects coefficient is lower than the group means given before which indicates a 1.2 percentage point increase in CPI for a 10 percentage point increase in PPI. The second model includes a lagged PPI term to control for dynamics. The coefficient of this term is rather small but statistically significant. The combined effect is higher than the result obtained in the first model but still lower than group means of individual OLS regressions. Using the residuals from the second model we computed the general cross sectional dependence test of Pesaran (2004). The test statistic which follows a standard normal distribution is 14.686 ($p < 0.00$). This shows that we can reject the null of no cross sectional dependence. Therefore, the results given above may be statistically invalid.

Table 2: Time Domain CCE Estimates

	$\hat{\beta}$	t-ratio		$\hat{\beta}$	t-ratio
Austria	0.003	0.044	Ireland	0.022	0.674
Belgium	-0.029	-0.854	Italy	0.020	0.433
Canada	0.157	4.053	Luxembourg	-0.023	-1.326
Chile	0.168	7.630	Netherlands	0.020	0.566
Czech Republic	0.071	1.843	Norway	0.079	2.014
Denmark	0.007	0.139	Poland	0.090	2.536
Estonia	0.233	3.357	Slovak Republic	0.069	1.517
Finland	0.126	2.642	Slovenia	-0.076	-0.694
France	-0.058	-1.496	Spain	0.248	3.797
Germany	0.118	0.995	Sweden	0.050	1.390
Greece	-0.056	-1.374	Switzerland	0.163	2.352
Hungary	-0.005	-0.145	United States	0.251	9.441
CCEMG: 0.068 (3.467)					

Note: Dependent variable is CPI. t-ratios are computed using bootstrap standard errors.

In the cross country price equations as the one above, it is reasonable to consider the common factors affecting each country. These common factors can be observable, like changes in oil prices, or unobservable such as global technology shocks which reduce the production costs. Therefore, these common factors will create dependence across countries in the panel and, if they are correlated with the explanatory variable in the model which is the case of PPI the consistency results of standard fixed effects model will not hold. To see the effect of these unobserved common factors on the brief panel data results given above, we computed the CCE and CCEMG estimates for the sample in hand. The results of these estimates are given in Table 2. As can be seen, for most of the countries individual parameter estimates are lower than the OLS estimates. For example, the estimate is 0.23 for Estonia for which we had found one of the biggest parameter estimates in the individual OLS regressions. In some extreme cases the difference between the two estimates is around 0.25, such as Slovenia, France, Germany. For Poland, Hungary and Czech Republic this difference is negative, which indicates that OLS estimates are bigger. However, for these countries, either parameter estimates are statistically insignificant in both regressions or they are small in magnitude. CCEMG is estimated as 0.068, which is around one half of the fixed effects estimates and it is economically and statistically insignificant.

Table 3: Band Spectrum CCE Regression

	Low Frequency		High Frequency		
	$\hat{\beta}$	t-ratio	$\hat{\beta}$	t-ratio	t-ratio
Austria	0.112	4.052	-0.083	-11.340	1.037
Belgium	-0.088	-32.443	0.031	12.394	-1.657
Canada	0.208	55.341	0.067	19.220	1.652
Chile	0.221	155.647	0.130	86.620	1.679
Czech Republic	0.202	21.968	-0.009	-2.080	1.831
Denmark	-0.133	-4.275	-0.055	-5.869	-0.388
Estonia	0.536	40.969	0.021	2.287	3.419
Finland	0.100	9.656	0.113	22.333	-0.103
France	-0.081	-3.941	0.159	21.516	-1.423
Germany	-0.020	-0.124	0.141	2.134	-0.337
Greece	0.007	0.277	-0.116	-6.115	0.578
Hungary	-0.042	-12.495	0.084	27.816	-1.578
Ireland	0.010	1.270	0.063	39.065	-0.554
Italy	0.053	5.281	0.036	5.442	0.133
Luxembourg	-0.039	-4.870	0.016	2.921	-0.478
Netherlands	-0.056	-9.272	0.103	19.295	-1.481
Norway	0.066	9.650	0.045	13.154	0.200
Poland	0.160	39.324	-0.014	-3.863	1.979
Slovak Republic	-0.160	-13.390	0.091	16.954	-1.909
Slovenia	-0.163	-4.152	-0.120	-4.422	-0.166
Spain	0.437	7.088	-0.521	-12.172	2.949
Sweden	-0.034	-3.302	0.078	26.431	-0.969
Switzerland	0.203	8.302	0.308	20.858	-0.531
United States	0.441	182.559	0.181	139.896	4.294
CCEMG:	0.0807	2.075	0.03111	1.0034	0.925

Note: Dependent variable is CPI. Zero frequency is always excluded to control for fixed effects. t-ratios are computed using bootstrap standard errors. They follow t-distribution with 96 degrees of freedom. Last column is the t-statistic for the null hypothesis of equality of low and high frequency estimates.

As shown before if the parameters of common factors are frequency dependent one must use band spectrum estimators for these parameters. And even if the common factors do not have a dynamic structure but the slope parameters are frequency dependent it is still a necessity to use the band spectrum estimators. The results for frequency domain versions of CCE and CCEMG are given in Table 3. As can be seen, the evidence on the sign of the difference between low and high frequency estimates is mixed: while for some of the countries low frequency estimates are bigger for the others they are smaller. An extreme case is Spain, where low frequency estimate is 0.43, the high frequency estimate is -0.52. And for instance, for Slovak Republic this difference is -0.25. While we can draw such an economic conclusion on the parameters, the differences between low and high frequency parameters are not usually statistically significant except the cases of Estonia, Spain and United States. The low frequency mean group estimate is 0.08 and high frequency mean group estimate is 0.031. This result again shows that on average, low frequency effect of PPI on CPI is bigger than the high frequency effect.

5 Conclusion

It is becoming increasingly popular to use panel data for investigating the time series behaviour of economic variables. This is especially because of the expectation to gain power in unit root and cointegration tests using different realizations of a time series. However, panel data has its own characteristics which can create some analysis difficulties. An example is cross sectional dependence. In this paper, following Pesaran (2006), we used a model with dynamic multifactor error structure we showed that his findings can be used in frequency domain analysis. Namely, we showed that the DFT of the cross sectional means of dependent variable and regressors can be used as proxies of unobserved common factors. We state that, asymptotic results of Pesaran holds for the estimators of frequency dependent coefficients. Using monthly data from 24 OECD countries for the period 02:2005 and 03:2014 we applied the method to price equations. After a descriptive analysis, for comparison purposes we ran time domain OLS regressions and individual band spectrum regression. We conclude that after controlling for unobserved factors both time domain and frequency domain parameter estimates are lower than the usual findings in the literature.

References

Anderson, T. W., and C. Hsiao (1981): "Estimation of dynamic models with error components," *Journal of the American statistical Association*, 76(375), 598–606.

Arellano, M., and S. Bond (1991): "Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations," *The review of economic studies*, 58(2), 277–297.

Bai, J. (2003): "Inferential theory for factor models of large dimensions," *Econometrica*, 71(1), 135–171.

Bai, J., and S. Ng (2002): "Determining the number of factors in approximate factor models," *Econometrica*, 70(1), 191–221.

Balestra, P., and M. Nerlove (1966): "Pooling cross section and time series data in the estimation of a dynamic model: The demand for natural gas," *Econometrica*, 34(3), 585–612.

Baltagi, B. (2008): *Econometric analysis of panel data*. John Wiley & Sons.

Banerjee, A., and J. L. Carrion-I-Silvestre (2006): "Cointegration in panel data with breaks and cross-section dependence," *European University Institute Working Papers, ECO No. 2006/5.*

Barro, R. J. (1991): "Economic growth in a cross section of countries," *The quarterly journal of economics*, 106(2), 407–443.

Beckett, L., and P. Diaconis (1994): "Spectral analysis for discrete longitudinal data," *Advances in Mathematics*, 103(1), 107–128.

Beggs, J. J. (1986): "Time series analysis in pooled cross-sections," *Econometric Theory*, 2(03), 331–349.

Blundell, R., and S. Bond (1998): "Initial conditions and moment restrictions in dynamic panel data models," *Journal of econometrics*, 87(1), 115–143.

Bond, S., A. Hoeffler, and J. Temple (2001): *GMM estimation of empirical growth models*, vol. 3048. Centre for Economic Policy Research London.

Breitung, J., and S. Das (2005): "Panel unit root tests under cross-sectional dependence," *Statistica Neerlandica*, 59(4), 414–433.

Brillinger, D. R. (2001): *Time series: data analysis and theory*, vol. 36. Siam.

Caporale, G. M., M. Katsimi, and N. Pittis (2002): "Causality links between consumer and producer prices: some empirical evidence," *Southern Economic Journal*, pp. 703–711.

Caselli, F., G. Esquivel, and F. Lefort (1996): "Reopening the convergence debate: a new look at cross-country growth empirics," *Journal of economic growth*, 1(3), 363–389.

Coakley, J., A.-M. Fuertes, and R. Smith (2002): "A principal components approach to cross-section dependence in panels," in *10th International Conference on Panel Data, Berlin, July 5-6, 2002*, no. B5-3. International Conferences on Panel Data.

——— (2006): "Unobserved heterogeneity in panel time series models," *Computational Statistics & Data Analysis*, 50(9), 2361–2380.

Coakley, J., and F. Kulasi (1997): "Cointegration of long span saving and investment," *Economics Letters*, 54(1), 1–6.

Colclough, W. G., and M. D. Lange (1982): "Empirical evidence of causality from consumer to wholesale prices," *Journal of Econometrics*, 19(2), 379–384.

Conley, T. G. (1999): "GMM estimation with cross sectional dependence," *Journal of econometrics*, 92(1), 1–45.

Corbae, D., S. Ouliaris, and P. C. Phillips (1994): "A reexamination of the consumption function using frequency domain regressions," *Empirical Economics*, 19(4), 595–609.

——— (1997): *Band spectral regression with trending data*. College of Business Administration. Department of Economics. University of Iowa.

——— (2002): "Band spectral regression with trending data," *Econometrica*, 70(3), 1067–1109.

Driscoll, J. C., and A. C. Kraay (1998): "Consistent covariance matrix estimation with spatially dependent panel data," *Review of economics and statistics*, 80(4), 549–560.

Engle, R. F. (1974): "Band spectrum regression," *International Economic Review*, 15(1), 1–11.

——— (1978): "Testing price equations for stability across spectral frequency bands," *Econometrica: Journal of the Econometric Society*, pp. 869–881.

——— (1980a): "Exact maximum likelihood methods for dynamic regressions and band spectrum regressions," *International Economic Review*, 21(2), 391–407.

——— (1980b): "Hypothesis Testing in Spectral Regression; the Lagrange Multiplier Test as a Regression Diagnostic," in *Evaluation of Econometric Models*, pp. 309–321. Academic Press.

Engle, R. F., and R. Gardner (1976): "Some finite sample properties of spectral estimators of a linear regression," *Econometrica: Journal of the Econometric Society*, pp. 149–165.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin (2000): "The generalized dynamic-factor model: Identification and estimation," *Review of Economics and statistics*, 82(4), 540–554.

——— (2005): "The generalized dynamic factor model: one-sided estimation and forecasting," *Journal of the American Statistical Association*, 100(471), 830–840.

Frankel, J. A., and A. K. Rose (1996): "A panel project on purchasing power parity: mean reversion within and between countries," *Journal of International Economics*, 40(1), 209–224.

Hannan, E. J. (1963a): "Regression for time series," in *Proceedings of the Symposium on Time Series Analysis*, ed. by M. Rosenblatt, New York. John Wiley and Sons Inc.

——— (1963b): "Regression for time series with errors of measurement," *Biometrika*, 50(3/4), 293–302.

——— (1965): "The estimation of relationships involving distributed lags," *Econometrica: Journal of the Econometric Society*, pp. 206–224.

——— (1967): "The estimation of a lagged regression relation," *Biometrika*, 54(3-4), 409–418.

Hannan, E. J., and R. Terrell (1973): "Multiple equation systems with stationary errors," *Econometrica*, 41(2), 299–320.

Ho, T.-w. (2002): "The Feldstein–Horioka puzzle revisited," *Journal of International Money and Finance*, 21(4), 555–564.

Holtz-Eakin, D., W. Newey, and H. S. Rosen (1988): "Estimating vector autoregressions with panel data," *Econometrica: Journal of the Econometric Society*, pp. 1371–1395.

Hsiao, C. (2003): *Analysis of panel data*, vol. 34. Cambridge university press.

Kapetanios, G., M. H. Pesaran, and T. Yamagata (2011): "Panels with non-stationary multifactor error structures," *Journal of Econometrics*, 160(2), 326–348.

Lee, J., and M. C. Strazicich (2003): "Minimum Lagrange multiplier unit root test with two structural breaks," *Review of Economics and Statistics*, 85(4), 1082–1089.

MacDonald, R. (1996): "Panel unit root tests and real exchange rates," *Economics Letters*, 50(1), 7–11.

Nickell, S. (1981): "Biases in dynamic models with fixed effects," *Econometrica: Journal of the Econometric Society*, pp. 1417–1426.

Oh, K.-Y. (1996): "Purchasing power parity and unit root tests using panel data," *Journal of International Money and Finance*, 15(3), 405–418.

Pesaran, M. H. (2004): "General diagnostic tests for cross section dependence in panels," *CESifo working paper series*.

——— (2006): "Estimation and inference in large heterogeneous panels with a multifactor error structure," *Econometrica*, 74(4), 967–1012.

Pesaran, M. H., T. Schuermann, and S. M. Weiner (2004): "Modeling regional interdependencies using a global error-correcting macroeconometric model," *Journal of Business & Economic Statistics*, 22(2), 129–162.

Pesaran, M. H., and E. Tosetti (2011): "Large panels with common factors and spatial correlation," *Journal of Econometrics*, 161(2), 182–202.

Phillips, P. C., and D. Sul (2003): "Dynamic panel estimation and homogeneity testing under cross section dependence*," *The Econometrics Journal*, 6(1), 217–259.

Phillips, P. J. B. (1991): "Spectral regression for cointegrated time series," in *Nonparametric and semiparametric methods in economics and statistic*, ed. by J. P. W. Barnett, and G. Tauchen. Cambridge University Press., Cambridge.

Sims, C. A. (1972): "Are there exogenous variables in short-run production relations," in *Annals of Economic and Social Measurement, Volume 1, number 1*, pp. 16–35. NBER.

Stock, J. H., and M. W. Watson (2002): "Forecasting using principal components from a large number of predictors," *Journal of the American statistical association*, 97(460), 1167–1179.

Westerlund, J. (2008): "Panel cointegration tests of the Fisher effect," *Journal of Applied Econometrics*, 23(2), 193–233.

Whittle, P. (1951): *Hypothesis testing in time series analysis*. Almqvist and Wicksell, Uppsala, Sweden.

Wu, Y. (1996): "Are real exchange rates nonstationary? Evidence from a panel-data test," *Journal of Money, Credit and Banking*, pp. 54–63.

A Summary Statistics and Time Series Regressions

Table 4: Summary Statistics

	CPI				PPI			
	Mean	Std. Dev.	Min.	Max.	Mean	Std. Dev.	Min.	Max.
Austria	0.17	0.36	1.18	-0.82	0.11	0.36	1.12	-1.01
Belgium	0.18	0.29	0.94	-0.62	0.28	1.05	2.37	-4.86
Canada	0.16	0.39	1.15	-1.04	0.18	0.85	1.94	-2.87
Chile	0.29	0.47	1.49	-1.30	0.43	1.47	4.48	-3.70
Czech Republic	0.20	0.49	2.97	-0.61	0.06	0.71	2.36	-2.72
Denmark	0.17	0.40	1.27	-0.50	0.19	0.46	1.13	-1.32
Estonia	0.34	0.46	2.17	-0.71	0.25	0.48	1.86	-1.50
Finland	0.17	0.32	1.05	-0.68	0.12	0.68	1.73	-2.75
France	0.13	0.32	0.83	-0.56	0.11	0.52	1.26	-1.80
Germany	0.14	0.32	0.81	-0.60	0.10	0.31	0.69	-1.23
Greece	0.20	1.21	3.31	-1.65	0.30	1.38	3.21	-4.91
Hungary	0.36	0.52	2.48	-0.57	0.25	1.14	3.51	-2.28
Ireland	0.13	0.49	1.25	-1.70	-0.01	0.91	2.93	-2.70
Italy	0.16	0.20	0.58	-0.36	0.15	0.54	1.28	-2.26
Luxembourg	0.20	0.69	1.90	-1.15	0.14	1.18	3.42	-3.89
Netherlands	0.16	0.47	1.19	-1.07	0.27	1.34	3.07	-5.81
Norway	0.17	0.48	1.60	-1.27	0.38	1.13	3.20	-3.29
Poland	0.22	0.34	1.22	-0.49	0.12	0.69	2.62	-1.72
Slovak Republic	0.22	0.37	2.05	-0.29	-0.04	0.73	1.85	-3.37
Slovenia	0.21	0.61	1.60	-1.14	0.14	0.37	1.29	-0.98
Spain	0.19	0.58	1.44	-1.33	0.19	0.64	1.40	-2.54
Sweden	0.11	0.44	1.03	-1.34	0.14	0.56	1.25	-1.15
Switzerland	0.04	0.40	1.13	-0.84	0.04	0.32	0.94	-0.89
United States	0.20	0.45	1.22	-1.92	0.27	1.08	2.34	-4.19

Table 5: OLS and BS Regressions

			Low Frequency		High Frequency		
	$\hat{\beta}$	t-ratio	$\hat{\beta}$	t-ratio	$\hat{\beta}$	t-ratio	t-ratio
Austria	0.215	3.189	0.3648	2.2132	0.0761	0.4788	1.2603
Belgium	0.152	6.913	0.1544	3.7732	0.1493	3.3256	0.0837
Canada	0.233	7.116	0.2796	4.3019	0.183	2.7348	1.0356
Chile	0.198	8.689	0.2384	5.8174	0.1298	2.4453	1.6203
Czech Republic	0.061	1.280	0.0105	0.0769	0.0826	0.9293	-0.4442
Denmark	0.194	4.663	0.2662	1.7441	0.1478	1.2137	0.6062
Estonia	0.396	4.477	0.7225	4.807	0.0022	0.0133	3.2266
Finland	0.199	6.079	0.2154	3.0808	0.1763	2.1836	0.3662
France	0.207	6.490	0.2384	3.0395	0.1393	1.1994	0.7075
Germany	0.375	5.974	0.2923	2.2065	0.5675	2.8061	-1.1384
Greece	0.088	3.137	0.1232	1.0035	0.0457	0.338	0.424
Hungary	-0.012	-0.313	-0.0833	-0.9689	0.0373	0.5214	-1.0783
Ireland	0.043	0.970	0.0708	0.5218	0.0366	0.5511	0.2264
Italy	0.167	6.008	0.205	3.6825	0.0754	0.875	1.2641
Luxembourg	0.056	2.491	0.0905	1.1696	0.0017	0.0172	0.7159
Netherlands	0.063	3.646	0.0373	0.8209	0.1128	1.7928	-0.9731
Norway	0.109	3.426	0.1277	1.6578	0.0988	1.6847	0.2983
Poland	0.087	2.234	0.1924	2.0875	0.0136	0.1759	1.4884
Slovak Republic	0.145	3.727	0.1426	1.6989	0.1481	1.6085	-0.0443
Slovenia	0.194	1.593	0.3035	1.3742	-0.0099	-0.0328	0.8403
Spain	0.365	12.199	0.356	3.1315	0.389	2.1585	-0.1547
Sweden	0.101	1.960	0.159	0.9563	0.0819	0.8638	0.4029
Switzerland	0.394	5.059	0.4942	2.7314	0.2911	1.5921	0.7896
United States	0.319	18.515	0.3541	6.713	0.2819	5.2735	0.9615
Group Means	0.181	7.534	0.223	6.529	0.136	4.951	2.3207

Note: Dependent variable is CPI. For OLS regressions degrees of freedom is 97, for band spectrum regression it is 96. Last column is the t-statistic for the null hypothesis of equality of low and high frequency estimates.